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Corollary II. The probability p that of n points, distributed at random on a closed curve of length a, no two fall within an arc b of each other is

$$p=\left(\frac{a-nb}{a}\right)^{n-1}.$$

For let one point be located at random and let the arc be cut at that point and straightened. No point which falls within a distance b of either end of this line gives a favorable case. We may therefore consider the favorable cases in the distribution of the remaining $n-1$ points on the line of length $a-2b$. The probability therefore involves one less integral sign and the limit of variation is not a but $a-2b$. The denominator however remains a, since this is the measure of the total number of possible cases. Replacing therefore n by $n-1$ in the expression P, and the a in the numerator by $a-2b$ we obtain p.

Corollary III. The probability that of n points distributed at random on a closed curve of length a, at least two fall within a distance b of each other is $1-p$.

◆

NUMERALS FOR SIMPLIFYING ADDITION.

By R. A HARRIS, Washington. D. C.

Primitive numerals are composed of the sign for unity repeated as many times as there are units in the numbers represented. Systems in which all nine figures are primitive were used by the Egyptians in their hieroglyphic writing, the Phœnicians, the Babylonians in their cuneiform inscriptions, the early Romans, and probably the early Indians. By virtue of such a notation, the process of addition can be reduced to counting by ones. The meaning of the symbols being self-evident does not have to be learned.*

In many instances, the first two, three, or four numerals are essentially primitive while the others are more arbitrary or superficial; *e. g.*, the hieratic symbols of the Egyptians, the ordinary Roman numrals, the moderately early Indian numerals, and the numerals now used by the Chinese.

The Palmyrenes made use of two component signs in constructing the first nine numerals,—simple strokes representing the ones and a certain Y-like symbol the component five found in each of the numbers five, six, seven, eight, and nine. Here addition can be performed by counting by ones and by fives,—the fives, for convenience, being taken in pairs so that most of the counting is really by tens instead of by fives. The same is true of the Roman numerals of the republican period where IIII is written for IV and VIIII for IX.

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*Columns 3, 5, 6, and 7 of the accompanying figure are copied from the Encyclopedia Britannica, article "Numerals."

The Syrians made use of three component signs; one for unity, another for two, and another for five. Here addition can be performed by counting by ones, twos, and fives (but for the most part, of course, by tens instead of fives).

Although our numerals underwent considerable change in form prior to the invention of the art of printing, there appears to be no doubt that the Europeans received them (zero included) from the Arabs, and that the Arabs received them from the Hindoos. The origin of these symbols is obscure. The most reasonable supposition seems to be that the significant digits (or at least those greater than three) were in the first place the initial letters of the Sanskrit words for the numbers which they represent.

Ingenious attempts have been made to show how each of these nine figures might have originally consisted of as many straight marks, conveniently joined together, as the number itself contains units; or how, instead of lines, angles were the quantities considered. History, however, does not confirm either of these hypotheses, although the first may apply to the figures one, two, and possibly three. But whatever their origin, our figures became, centuries ago, mere signs of numbers conveying no suggestion of decomposition into component parts.*

In the second column of the accompanying table is a proposed set of figures, in which the component ones are vertical strokes, the twos horizontal strokes, and the fives closed loops. These figures are fairly distinctive however turned, can be made with comparative ease, and possess the important property of being obviously composed of ones, twos, and fives, as were the Syrian characters.

The stroke constituting figure two has been slightly curved so as to better distinguish it from the minus sign or the dash, and to prevent its being mistaken for figure one when it happens to be turned into the vertical. The strokes in figure four are likewise curved to distinguish the latter from the sign of equality, and to prevent confusion between the symbols for four and eleven.

The figures one, six, and eight of the proposed system closely resemble their ordinary or Arabic forms.

A convenient rule for adding a column of moderate length may be given thus: Count by twos the horizontal marks (all terminating to the right); upon this sum count the ones, known by the vertical marks; upon this result count by fives the closed loops, generally by tens (the loops being taken in pairs. The result of these three counts is the sum of the column.

To add the next column (to the left), begin by counting by twos if the number carried be even. If it be odd, before proceeding mark a component one of the column to be added by placing a dot above it and count one upon the odd number carried thus making it even. Then proceed as in the first column, but taking care to omit the dotted component one when the ones are counted.

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*The following references are to works in which the history of numerals is considered:

Hankel: *Zur Geschichte der Mathematik in Altertum und Mittelalter.*

Brooks: *The Philosophy of Arithmetic*, pp. 21-31, 101-107, 141-146.

Encyclopedia Britannica, under "Numerals" and "Inscriptions."

Cantor: *Vorlesungen ueber Geschichte der Mathematik*, Vol. I, and Plate at end of Vol. I.

Fine: *The Number-System of Algebra*, pp. 79-90.

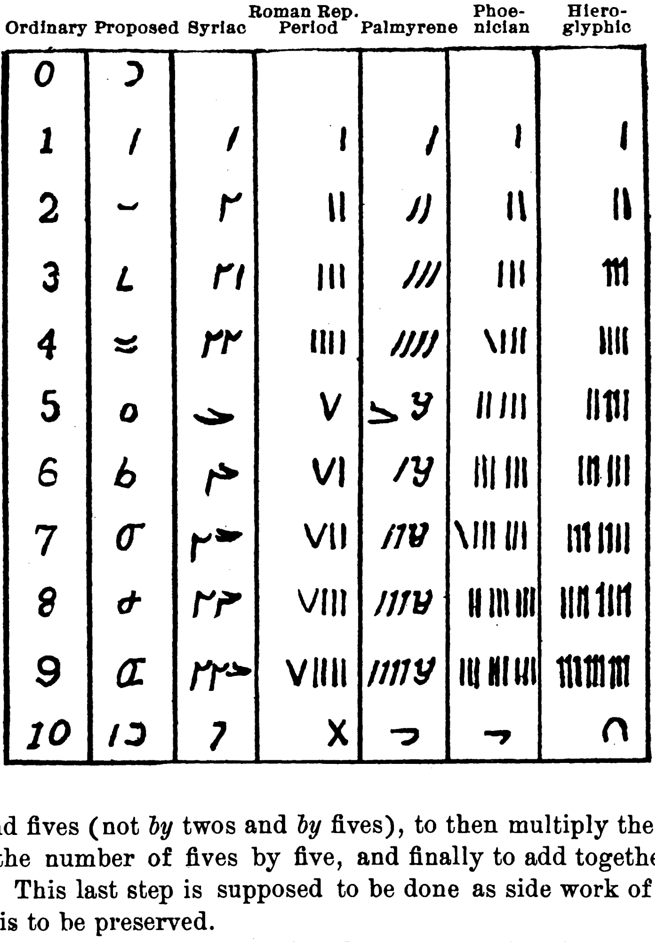
Ball: *A Short Account of the History of Mathematics*, 2d Ed., pp. 123-131, 141, 158, 161, 189-191.

Fink: *A Brief History of Mathematics*, pp. 6-17.

Encyclopedie des Sciences Mathematiques, I, 1, 1, pp. 20-21.

If the person adding can count equally well the odd numbers by twos, or if there should happen to be no component one in the column to be added, the borrowing of a one will not be resorted to.

In the case of a very long column, it may be best to simply count the ones, twos, and fives (not *by* twos and *by* fives), to then multiply the number of twos by two, the number of fives by five, and finally to add together the three partial sums. This last step is supposed to be done as side work of which only the total sum is to be preserved.


Ordinary
Proposed
Syriac
Roman Rep. Period
Palmyrene
Phoenician
Hieroglyphic
0
1
2
3
4
5
6
7
8
9
10
I
II
III
IIII
V
VI
VII
VIII
VIIII
X

These numerals are proposed for the purpose of saving mental labor in performing addition. The time required in adding by the above methods is about the same as that required by the usual method, or possibly a little greater, but the mental strain is much less because of the symmetry of the process. The liability to mistakes is somewhat less and for the same reason.

Many subtractions are self-evident; *e. g.*, 87−52=35. Here the components of the subtrahend are simply stricken from the minuend, and the result of this cancellation is the difference required, *i. e.*, 87—52=35.

If characters like those proposed were in use, it seems probable that children could learn addition tables with far greater ease and certainty than is the case at present.

Finally, it may be remarked that if addition be carried out in the ordinary manner, these characters will be quite as serviceable as are the Arabic numerals, and being adapted to the mode of addition described above, a new and convenient way of proving the work will be afforded.

◆

THE CONVEX SURFACE OF AN OBLIQUE CONE.

By F. P. MATZ, Reading, Pa.

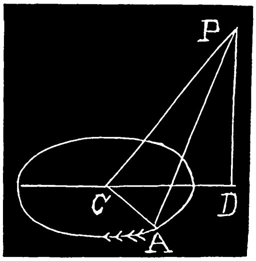
Legendre, in his *Théorie des fonctions elliptiques*, has devised a method for finding the convex surface of an oblique cone having a circular base.

We propose the following method: Let $CA=r$, $PD=a$, and $\angle PCD=\omega$. Represent the uniformly varying $\angle DCA$ by θ; then since $CD=a\cot\omega$, we have by trigonometry,

$$\cos\theta=\frac{a^2\cot^2\omega+r^2-(AD)^2}{2ar\cot\omega}\text{.........}(1).$$

$$\therefore AD=\sqrt{}(a^2\cot^2\omega+r^2-2ar\cot\omega\cos\theta)\text{.........}(2),$$

and $AP=\sqrt{}(a^2\operatorname{cosec}^2\omega+r^2-2ar\cot\omega\cos\theta)$ .... (3),



which *line* represents an element of convex surface of the oblique cone.

$$\therefore S=2r\int_0^{\pi}\sqrt{}(a^2\operatorname{cosec}^2\omega+r^2-2ar\cot\omega\cos\theta)d\theta\text{..........}(4).$$

Transforming (4) under the supposition that $\theta=\pi+2\phi$, we get

$$S=2r\int_0^{\frac{1}{2}\pi}\sqrt{}(a^2\operatorname{cosec}^2\omega+r^2+2ar\cot\omega\cos2\phi)\times2d\phi\text{.........}(5),$$

$$=4r\int_0^{\frac{1}{2}\pi}\sqrt{}(a^2\operatorname{cosec}^2\omega+r^2+2ar\cot\omega-2ar\cot\omega\sin^2\phi)d\phi\text{..........}(6),$$

$$=4r\int_\theta^{\frac{1}{2}\pi}\sqrt{\left[\left(\frac{a^2+2ar\sin\omega\cos\omega+r^2\sin^2\omega}{\sin^2\omega}\right)-\left(\frac{2ar\sin\omega\cos\omega}{\sin^2\omega}\right)\sin^2\phi\right]}d\phi\text{.........}(7).$$